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$x=3.131312,518250,572965,804300,733409,211483,7\pm$

$y=-2.805118,086952,744853,053572,398087,397376,0\mp$

$x_{11}=-3.283185,991286,169412,266000,514372,745305,5\pm$

$y_{11}=-3,779310,253377,746891,890765,841292,764578,6\mp$

$*S^3\mp_1 2xS^2-2yS=\mp f\ldots.(J)$ by $y=\mp_1 Sx+\frac{1}{2}(S^2\pm_1\Sigma)\ldots.(K)$ in (I).

$\therefore x^2\mp_1 Sx=7-\frac{1}{2}(S^2\pm_1\Sigma)$ give by quadratics all x's and y's with no further aid.

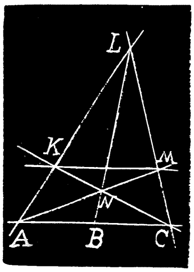
GEOMETRY.

102. Proposed by B. F. FINKEL, A. M., M. Sc., Professor of Mathematics and Physics, Drury College, Springfield, Mo.

Ohne Benutzung des Cirkels eine Strecke AC zu halbiren, wenn eine Parallele der Geraden AC gegeben ist. [Reye's Geometrie der Lage, Part I, page 191].

I. Solution by C. HORNUNG, Professor of Mathematics, Heidelberg University, Tiffin, Ohio.

Wir ziehen durch irgend einen Punkt L auserhalb der gegebenen Parallele die Linien AL und CL welche die Parallele in resp. K und M schneiden. Dann ziehen wir die Linien AM und CK die sich im Punkte N schneiden. So bestimmt die Diagonale LN des Vierecks $KLMN$ den gesuchten Halbirungspunkt B der Strecke AC. Denn der vierte von B harmonisch getrennte Punkt, in welchem die zweite Diagonale KM des Vierecks die Gerade ABC schneidet, liegt uendlich fern.



II. Solution by the PROPOSER.

GIVEN the line AC and the line KM parallel to it,

TO BISECT AC without using the circle.

CONSTRUCTION. 1. Through A draw any two lines intersecting the parallel line in the points K and M.

2. Draw the lines CK and CM, CK intersecting AM in N and CM intersecting AK in L.

3. Draw the diagonal, LN, of the 4-side, $LMNK$, and its intersection B with AC will be the required point of bisection.

The points A, B, C, and the point at infinity, *i. e.* the intersection of AC and MK, constitute a harmonic range of points and the point at infinity is harmonically separated from B by the points A and C.

This problem was also solved by *G. B. M. ZERR*, *C. C. CROSS*, *J. SCHEFFER*, *A. F. KOVARIK*, *M. A. GRUBER*, *NELSON L. RORAY*, *ELMER SCHUYLER*, and *JOHN MANIE*.

103. Proposed by FREMONT CRANE, Sand Coulee, Mont.

A horse is tethered with a rope which is attached to a stake B on the edge of a circular pond containing one acre. How long must the rope be to allow the horse to graze over one acre? [From *Home Study Magazine*, problem 249].

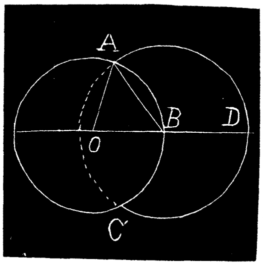
I. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.; COOPER D. SCHMITT, A. M., Professor of Mathematics, University of Tennessee, Knoxville, Tenn.; P. S. BERG, B. Sc., Principal of Schools, Larimore, N. D.; WALTER H. DRANE, Graduate Student, Harvard University, and M. A. GRABER, Tiffin, Ohio.

Let B be the point where the horse is tethered, A and C the points on the edge of the given pond, to which the horse can graze, O the center of the pond. Let $AO=r=4\sqrt{(10/\pi)}$, the radius of the pond, and $\angle ABO=\theta$.

$\therefore AB=$length required$=2r\cos\theta$. Also the area common to the pond and the circle upon which the horse can graze$=r^2(\pi+2\theta\cos2\theta-\sin2\theta)$.

$\therefore$ The area upon which the horse can graze outside the pond is $4\pi r^2\cos^2\theta-r^2(\pi+2\theta\cos2\theta-\sin2\theta)=\pi r^2$. $\therefore 2\theta-\tan2\theta=2\pi$.

$\therefore \theta=51^\circ\ 16'\ 24''$. $\therefore AC=2r\cos\theta=8.92926$ rods.



II. Solution by A. H. BELL, Hillsboro, Ill.

Let $10a=1$ acre, be the area of the pond, $10b$ the area grazed over, $R=\sqrt{(10a/\pi)}$ be its radius, and $AB=r=2R\sin\theta$ be the length of rope, where $2\theta=\angle AOB$. Then $r^2=4R^2\sin^2\theta=20a(1-\cos2\theta)/\pi$......(1).

2 sector $ABD-2$ segment $AB=10b$, or $r^2(\pi+2\theta)-2R^2(2\theta-\sin2\theta)=20b$....(2).

Substituting (1) in (2) and letting $\cos2\theta=x$ and $20b\pi/20a=c$, we get $2\theta x=\pi-c-\pi x+\sqrt{(1-x^2)}$, or

$$2\theta x=\tfrac{1}{2}\pi-c-\pi x+1-\tfrac{1}{2}x^2-x^4/2.4-3x^6/2.4.6-\text{ etc....(3)}.$$

By trigonometry,

$$2\theta x=(\tfrac{1}{2}\pi)x-x^2-x^4/1.2.3-3^2x^6/1.2.3.4.5.6-\text{ etc....(4)}.$$

(4)−(3) and transposing,

$$(\pi-c)+1=y=(3\pi/2)x-\tfrac{1}{2}x^2-x^4/2.3.4-x^6/4^2.5-\text{ etc....(5)}.$$

Assume $\cos2\theta=x=ay+by^2+cy^3+dy^4+$ etc....(6).

Substituting (6) in (5) and equating the coefficients of like powers of y, we have $a=2/3\pi$, $b=4/27\pi^3$, $c=16/243\pi^5$, $d=(80+12\pi^2)/(2187\pi^7)$, etc.

Hence, $\cos2\theta=(2y/3\pi)+(4y^2/27\pi^3)+(16y^3/243\pi^5)+[(80+12\pi^2)y^4]/(2187\pi^7\pi)$ +etc. But $c=\pi$, and $y=1$.

$\cos2\theta=.212207+.004778+.000215+.000030+....=.21723$.

Hence $2\theta=77^\circ\ 27'\ 12'$.

From (1) $r=2R\sin\theta=8.92$ rods.

III. Solution by CHARLES C. CROSS, Libertytown, Md.

Let $a=$length of rope, $b=$diameter of pond, and $A=$area over which the horse can graze. Then $(a^2/6b)(4a+3\pi b)=A$. [See Vol. IV, No. 1, of MONTHLY].

But by the problem $b=2\sqrt{(A/\pi)}$.

Substituting and reducing, we have,

$$a^3+\tfrac{3}{2}\sqrt{(\pi A)}a^2-3\sqrt{(A^3/\pi)}=0.$$

Restoring numbers, $a^3+33.62a^2-3425.514=0$,

whence $a=8.933+$ rods.